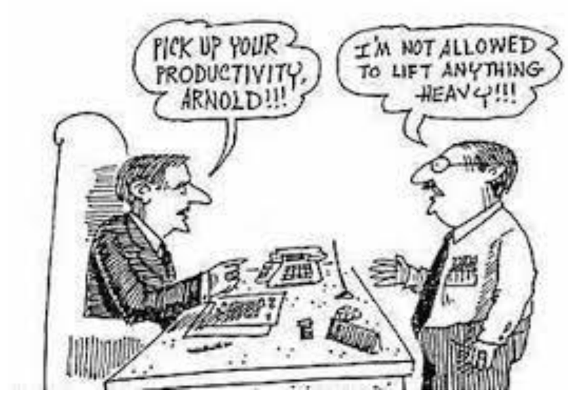
Productivity

John Van Reenen

Coase Initiative: Masterclass in Productivity,

Chicago, May 2023



THE LONDON SCHOOL OF ECONOMICS AND POLITICAL SCIENCE

Programme on Innovation and Diffusion

SOME KEY READINGS

Ackerberg, Daniel, Lanier Benkard, Stephen Berry and Ariel Pakes (2007) "Econometric tools for analyzing market outcomes" *Handbook of Econometrics* Chapter 63 Volume 6 (Editors: Leamer and Heckman). Part 2 on Production functions

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Gandhi, Amit, Salvador Navarro, and David A. Rivers. 2020. "On the identification of gross output production functions." *Journal of Political Economy* 128 (8): 2973–3016.

Griliches and Mairesse (1998), "Production Functions: The Search for Identification" Chapter 6 in *Econometrics and Economic Theory in the 20th Century: The Ragnar Frisch Centennial Symposium*

Olley, Steven and Ariel Pakes (1996) "The Dynamics of Productivity in the Telecommunications Equipment Industry", *Econometrica* 64(6), 1263–1298.

Syverson, Chad (2011) "What determines productivity?" *Journal of Economic Literature*, 49(2) 326-65

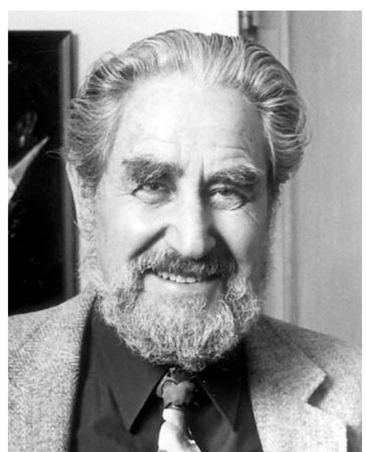
- **Why estimate firm production functions?**

- Productivity Facts

- How to estimate production functions

ESTIMATING PRODUCTION FUNCTIONS

- Estimation of production functions has a long history closely related to agriculture, panel data and **weather shocks**
  - Von Thuenen collected & recorded data on his northern German farm 1820-30, formed basis of his theories
  - US Dept. of Agriculture (Tolley et al, 1924; Moore, 1929; Cassels, 1936). Cobb and Douglas (1928)
  - Major econometric advances we'll discuss start with Hoch (1955) and Mundlak (1961) who introduced fixed effect models



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- Recent Interest revitalized by availability of micro-panel data on firms – establishments from US Census Bureau & company accounts like Compustat & BVD ORBIS

Relevance for Environmental Economics

1. **What is impact of environment on productivity?**
   - Environmental shocks (climate, floods, etc.)
   - Policy interventions (e.g. environmental regulation)
   - Need to measure productivity first, in particular taking into account other inputs (TFP)

2. **Measuring resource efficiency in particular**
   - Direction of technical change (clean vs. dirty)
   - Measure firm resource efficiency (output elasticity wrt materials)
     - Market reallocates to the more profitable firms, not necessarily those who are most resource efficient

General Relevance

1. **Original questions:**
   - How big are (dis)economies of scale?
   - Are factors of production paid their marginal Products (e.g. monopsony power over labor)?
   - How large are price-cost markups (measuring marginal costs)

General Relevance

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    - How large are price-cost markups (measuring marginal costs)
2. Much of growth **within** countries & productivity differences **across** countries seems related to TFP
3. Persistent Productivity Differentials across firms is very large & this is important for macro changes via reallocation
4. Important impact of policies on productivity, e.g. **Trade Opening** (e.g. Pavcnik, 2002; Goldberg et al, 2016; Bloom, Draca & Van Reenen, 2016); **Deregulation** (e.g. Olley-Pakes, 1996); **FDI** (e.g. Amiti et al, 2023), etc.

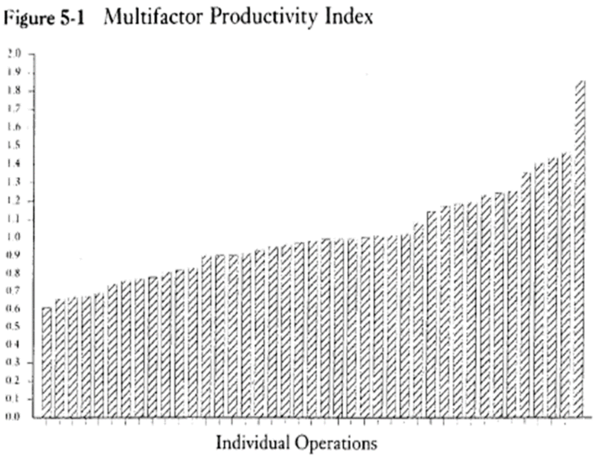
- Why estimate firm production functions?
- **Productivity Facts**
- How to estimate production functions

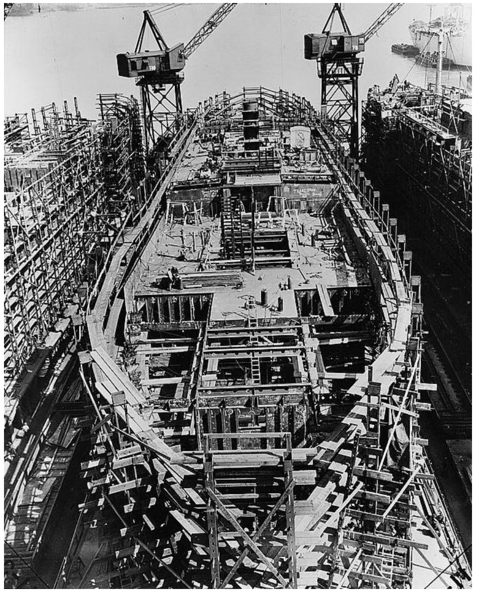
Productivity Heterogeneity a classic issue

- **Salter (1960)** 1911-26 UK pig-iron industry: best factory twice as much output per worker as average factory

- **Argote et al (1990)** US Liberty ships in WW2 across 16 separate shipyards. Focuses on learning by doing but also big yard-specific effects

- **Chew et al (1990)** 40 operating units in commercial food division of large US corporation. Top ranked unit twice as productive as bottom ranked (within firm)





Productivity Heterogeneity: basic facts

- Typical gap between 10th and 90th percentiles within same US four-digit industry (Syverson, 2004, 2011)
  - Labor Productivity (output per worker) 4:1 ratio
  - Total Factor Productivity 2:1 ratio

- Productivity dispersion generally larger in other countries
  - Hsieh & Klenow (2009) China and India 5:1 ratio
  - Bartelsman, Haltiwanger & Scarpetta (2013) 9 OECD countries.
  - And now replications in very many countries: OECD Multiprod data initiative (Criscuolo et al, 2016); ECB CompuNet; World Bank, etc.

- About half of aggregate TFP growth related to reallocation (~100% in US retail: Foster, Haltiwanger & Kirzan, 2006)

Are TFP differences "just measurement error?"

- **Parallels old debate in macro growth accounting**
  - Solow (1957) vs. Griliches & Jorgensen (1967)
- **It's <u>not</u> all measurement error:** as measured TFP correlated with future **survival** & **growth** as dynamic models suggest

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- **It's <u>not</u> all measurement error:** as measured TFP correlated with future **survival** & **growth** as dynamic models suggest
- But important issues

Some types of measurement error

1. *Measurement:* **capital** (want replacement value but often historic accounting value; not 100% utilization; depreciation issues; types, etc.); **labor** (hours; composition); **intermediates** (B2B issues, quality)

2. *Missing input quantities (& prices):* e.g. intangible capital (like IP)

3. *Missing output prices* (focus of literature)

4. *Multiproduct firms*

5. *Other issues:* Recording Errors; Imputation (US Census in 2007 73% of obs have at least one TFP element imputed)

- Why estimate firm production functions?
- Productivity Facts
- **How to estimate production functions**

BASICS OF PRODUCTION FUNCTION ESTIMATION

- We want to estimate $Q_{it} = A_{it} F_t(K_{it}, L_{it})$

Q_{it} = value added (or output if include other inputs like materials or energy) of firm *i* at time t,

K = capital

L = labor

A = TFP ("Hicks-Neutral")

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Think of K as a (quasi) fixed input and L as a variable input (energy, materials)

BASICS OF PRODUCTION FUNCTION ESTIMATION

- Likely to be **omitted variables**.
  - *Example:* Mundlak (1961) "Empirical production function free of management bias."
    - Managerial ability correlated with factor choice and output. Unless we measure directly (e.g. Bloom & Van Reenen, 2007) we will have biased parameter estimates
  - **Endogeneity:** Factor inputs are chosen by the firm, (Marschak & Andrews, 1944)

The Endogeneity Problem

Can the economist measure the effect of changing the amounts of labor and capital on the firm's output - the "production function" - in the same way in which the agricultural research worker measures the effect of changing amounts of fertilizers on the plot's yield?

He [sic] cannot because the manpower and capital used by each firm [are] determined by the firm, not by the economist. ***This determination is expressed by a system of functional relationships; the production function, in which the economist happens to be interested, is but one of them.***

Marschak and Andrews (1944) p.144

COBB-DOUGLAS EXAMPLE

- $\ln Q_i = \ln A_i + \alpha_L \ln L_i + \alpha_K \ln K_i$
- $q_i = a_i + \alpha_L l_i + \alpha_K k_i + \varepsilon_i$
  - Where q = ln(Q), l = ln(L), k = ln(K)
  - ε_i an error term (unobserved shock)
- Returns to scale = $\alpha_L + \alpha_K$ (if $\alpha_L + \alpha_K = 1$, CRTS)

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- Returns to scale = $\alpha_L + \alpha_K$ (if $\alpha_L + \alpha_K = 1$, CRTS)
- If inputs all chosen before shock then estimate by OLS
  - But this is unlikely to hold. Some adjustment to shock esp. variable factors, to factors unobserved by econometrician
  - e.g. $\varepsilon_i = \omega_i + e_i$ where ω_i is average farm weather (affects inputs) & e_i is within year weather shocks (after input decisions made)
  - Sometimes called the "transmission problem" (Griliches & Mairesse, 1998) as ω_i shocks transmit to input choice

SOME SOLUTIONS TO THE PRODUCTION FUNCTION PROBLEM

1. Residual approach (Index number methods)
2. IV Approach I - “External instruments”
3. Fixed effects (FE)
4. IV+FE (Arellano & Bond; Blundell & Bond)
5. Olley Pakes (1996) control function & extensions:
   - Levinsohn & Petrin (2003); Ackerberg, Caves & Frazer (2015); Goldberg et al. (2016); Gandhi et al. (2020); Orr (2022); Rubens (2023); etc

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1. RESIDUAL APPROACH (Solow, 1957)

- Assume perfect competition in factor and product markets, so can replace parameters by factor shares in revenue
- e.g. FOC for labor, P= output price, W = wage
- $\alpha_L = (WL/PQ) = s_L$
- So TFP = $\ln A_i = \ln Q_i - \alpha_L \ln L_i - \alpha_K \ln K_i$
- If CRTS TFP $= \ln Q_i - s_L \ln L_i - (1-s_L)\ln K_i$
- Can relax many assumptions

1. RESIDUAL APPROACH – RELAXING ASSUMPTIONS

- **Example (1) Perfect Competition in Labor Market**
  - Wages determined by bargaining over wage (e.g. monopoly union model). FOC condition still holds.
  - But no longer true in other bargaining models (e.g. efficient contracting over wages & employment as in Leontief, 1944) or monopsony (Rubens, 2023)

1. RESIDUAL APPROACH – RELAXING ASSUMPTIONS

- **Example (2) Functional form**
- Translog Production function (C-D first order approximation to general production function, Y = AF(K,L); Translog 2nd order approximation)
- Use "Tornqvist Index"

$$\ln TFP_i = \ln(Y_i / \bar{Y}) - \left(\frac{s_{iL} + \bar{s}_L}{2}\right)\ln(L_i / \bar{L}) - \left(\frac{s_{iK} + \bar{s}_K}{2}\right)\ln(K_i / \bar{K})$$

- Express firm outputs & inputs (X_i) relative to a "reference firm" in the industry ($\bar{X}$) such as the average. Shares are an average of firm's own share and the reference firm

SOME SOLUTIONS TO THE PRODUCTION FUNCTION PROBLEM

1. Residual approach
2. **IV Approach I - “External instruments”**
3. Fixed effects (FE)
4. IV+FE (Arellano & Bond; Blundell & Bond)
5. Olley Pakes + extensions

2. IV APPROACH - "EXTERNAL INSTRUMENTS"

- Factor input prices are **candidate instruments**
  - These enter factor demand equations, but not (in neoclassical model) for production function
  - **Problem:** obtaining exogenous firm-specific variation. e.g. base interest rate common to all firms; firm specific wage variation could be due to rent-sharing or changes in unobserved composition

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- **Solution?** Seek exogenous variation in factor prices
  - *Labor:* Min. wages; union power; local labour market shocks (e.g. Doraszelski & Jaumandreu, 2018)
  - *Capital:* tax rates (e.g. Cummins et al, 2002; Chetty & Saez, 2005; Bloom, Schankerman & Van Reenen, 2013)
  - *Intermediates*: trade policy changes; Bartik approaches interacting import mix and exchange rate fluctuations
- These are context specific & hard to find (no "easy recipe")

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3. FIXED EFFECTS (FE)

- If we have panel data then estimate $\ln Q_{it} = \alpha_L \ln L_{it} + \alpha_K \ln K_{it} + \varepsilon_{it}$
- Assume "variance components" of ε_{it} so:
  - $\varepsilon_{it} = \eta_i + \tau_t + v_{it}$
  - **Fixed effects** = η_i ; **time dummies** = τ_t , v_{it} **idiosyncratic error** (i.e. uncorrelated with K_{it} and L_{it}), i.e. all factor inputs strictly exogenous
  - Then estimate by TWFE (Hoch, 1955; Mundlak, 1957)
  - Include full set of firm dummies ("within groups" – same as doing mean deviations) or take first (or longer) differences

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  - Include full set of firm dummies ("within groups" – same as doing mean deviations) or take first (or longer) differences
- **Problems:**
  - Makes classical measurement error worse (attenuation bias – see Griliches & Mairesse, 1998)
  - Doesn't deal with time-varying (correlated) shocks, e.g. $E(\ln L_{it} v_{it})>0$

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4. IV AND FIXED EFFECTS (Anderson-Hsiao, 1982)

- Assume adjustment costs so that capital and labour depend on past values, e.g. $K_t = h(K_{t-1}, K_{t-2}, \text{etc.})$
- First differences (Δ) eliminates fixed effects $\Delta\eta_i = 0$.
    - $\mathbf{\Delta lnQ_{it} = \alpha_L \Delta lnL_{it} + \alpha_K \Delta lnK_{it} + \Delta v_{it}}$
- Since $E(\Delta lnL_{it}\ \Delta v_{it}) \neq 0$ & $E(\Delta lnK_{it}\ \Delta v_{it}) \neq 0$, use lnL_{it-2} & lnK_{it-2} as instruments for ΔlnL_{it} & ΔlnK_{it}

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  - $$\Delta \ln Q_{it} = \alpha_L \Delta \ln L_{it} + \alpha_K \Delta \ln K_{it} + \Delta v_{it}$$
- Since $E(\Delta \ln L_{it}\, \Delta v_{it}) \neq 0$ & $E(\Delta \ln K_{it}\, \Delta v_{it}) \neq 0$, use $\ln L_{it-2}$ & $\ln K_{it-2}$ as instruments for $\Delta \ln L_{it}$ & $\Delta \ln K_{it}$
- IV valid because:
  - $\ln L_{it-2}$ & $\ln K_{it-2}$ will be correlated with factor inputs through adjustment costs, e.g. $\ln K_{it} - \ln K_{it-1} = \lambda(\ln K_{it-1} - \ln K_{it-2})$
  - $\ln L_{it-2}$ & $\ln K_{it-2}$ uncorrelated with current productivity changes: i.e. v_{it} not serially correlated (by assumption)
- See Bond & Söderbom (2005); Ackerberg et al (2015) Monte Carlo simulations

PROBLEMS WITH IV FIXED EFFECTS

- **Inefficient**
  - Current model is just identified (one IV per endogenous variable)
  - But under the timing assumptions, not only are $\ln L_{it-2}$ & $\ln K_{it-2}$ IVs, but so are $\ln L_{it-3}$ & $\ln K_{it-3}$, $\ln L_{it-4}$ & $\ln K_{it-4}$, etc.
  - As panel goes on can use these to construct a General Method of Moments (GMM) estimator (see Arellano and Bond, 1991)
- **Omitted Dynamics**
  - If v_{it} serially correlated this invalidates IV: $E(\ln L_{it-2}\, \Delta v_{it}) \neq 0$
  - Can deal with this so long as serial correlation is "finite". Use only longer lags as IV's
  - e.g. MA(1) like $v_{it} = u_{it} + \phi\, u_{it-1}$, just use $\ln L_{it-3}$ & $\ln K_{it-3}$ (and longer) as IVs

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- **Weak Instruments (Important in practice)**
  - If variable (e.g. capital) doesn't change much over time then lags will be *weak instruments* for changes in capital (e.g. If random walk hopeless). In finite sample weak IV causes bias.
  - Similar to problem of attenuation bias due to measurement error.

BLUNDELL & BOND (1998, 2000) MOMENTS TO TACKLE WEAK INSTRUMENT PROBLEM

- Initial conditions assumption: initial input & output growth uncorrelated with fixed effects. Can also use *lagged differences* to instruments l*evels* (Blundell & Bond, 1998)
  - **"System GMM":** uses system of two sorts of moment restrictions (lagged levels in difference equation like A-B AND lagged differences in a levels equation)

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  - **"System GMM":** uses system of two sorts of moment restrictions (lagged levels in difference equation like A-B AND lagged differences in a levels equation)
  - In empirical applications, tends to generate more sensible production function results (e.g. on capital)
  - But strong assumption. Violated if high productivity firms grow faster, as we think young firms will if there is reallocation (so a kind of stationarity assumption more applicable to mature firms/industries)

Blundell & Bond (2000) is application of Blundell & Bond (1998) to a production function context

- If v_{it} is AR(1), we get a dynamic production function with a "common factor" (COMFAC) restriction

BLUNDELL-BOND

- $q_{it} = \alpha_L l_{it} + \alpha_K k_{it} + \varepsilon_{it}$ (equation 1)
  - $\varepsilon_{it} = \eta_i + \tau_t + v_{it\,;}\, v_{it}$ is AR(1):
  - $v_{it} = \rho v_{it-1} + u_{it\,,}\, u_{it}$ ~i.i.d.

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- Lag: $q_{it-1} = \alpha_L l_{it-1} + \alpha_K k_{it-1} + \eta_i + \tau_{t-1} + v_{it-1}$ Pre-multiply by ρ
  - $\rho q_{it-1} = \alpha_L \rho l_{it-1} + \alpha_K \rho k_{it-1} + \rho\eta_i + \rho\tau_{t-1} + \rho v_{it-1}$ (equation 2)
- Deduct equation (2) from (1)
- $q_{it} = \rho q_{it-1} + \alpha_L l_{it} - \alpha_L \rho l_{it-1} + \alpha_K k_{it} - \alpha_K \rho k_{it-1}$

  $+(1-\rho)\eta_i + \tau_t - \rho\tau_{t-1} + v_{it} - \rho v_{it-1}$

  $= \pi_1 q_{it-1} + \pi_2 l_{it} + \pi_3 l_{it-1} + \pi_4 k_{it} + \pi_5 k_{it-1}$

  $+(1-\rho)\eta_i + \tau_t - \rho\tau_{t-1} + v_{it} - \rho v_{it-1}$

  Where $\pi_1 = \rho$; $\pi_2 \pi_1 = -\pi_3 = \rho\alpha_L$; $\pi_4 \pi_1 = -\pi_5 = \rho\alpha_K$

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  $+(1-\rho)\eta_i + \tau_t - \rho\tau_{t-1} + v_{it} - \rho v_{it-1}$

  Where $\pi_1 = \rho$; $\pi_2 \pi_1 = -\pi_3 = \rho\alpha_L$; $\pi_4 \pi_1 = -\pi_5 = \rho\alpha_K$
- So can use standard GMM with t-2 and earlier instruments (as error MA(1)) & impose COMFAC restriction

SOME SOLUTIONS TO THE PRODUCTION FUNCTION PROBLEM

1. Residual approach
2. IV Approach I - “External instruments”
3. Fixed effects (FE)
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5. OLLEY-PAKES (1996) AND "PROXY METHODS"

$$q_{it} = \alpha_L l_{it} + \alpha_K k_{it} + \omega_{it} + \nu_{it}$$

- **Idea:** Use a control function to proxy out persistent part of productivity (ω_{it}). OP use **investment** (INV_{it}):
  - $INV_{it} = i(\omega_{it}, k_{it})$; This kind of investment rule arises in many dynamic models, e.g. Pakes (1994)
- ω_{it} <u>known</u> to firm, but not econometrician; ν_{it} is <u>unknown</u> prior to input choice & purely transitory (i.i.d. over time)
- Assume investment chosen at t-1 determining capital at t, i.e. **Capital** evolves deterministically $k_{it} = i_{it-1} + (1-\delta)k_{it-1}$ where δ is depreciation rate and i = ln(INV)
- **Labor** is non-dynamic (only current labor matters)

OLLEY-PAKES

- OP assume:
  - ω_{it} follows first order Markov Process: $\omega_{it} = g(\omega_{it-1}) + \xi_{it}$; (e.g. AR(1))
  - Investment is strictly monotonic in ω_{it} : $i_{it} = i_t(\omega_{it}, k_{it})$

- Key idea is to invert investment rule so $\omega_{it} = h_t(i_{it}, k_{it})$.

- $q_{it} = \alpha_L l_{it} + \alpha_K k_{it} + h_t(i_{it}, k_{it}) + \nu_{it}$

- $q_{it} = \alpha_L l_{it} + \phi_t(i_{it}, k_{it}) + \nu_{it;}$
  - NOTE: $\omega_{it} = \phi_t - \alpha_K k_{it}$

OLLEY-PAKES: TWO STEP ROUTINE

- $q_{it} = \alpha_L l_{it} + \phi_t(i_{it}, k_{it}) + \nu_{it;}$ $\omega_{it} = \phi_t - \alpha_K k_{it}$ (OP2)

- **<u>First stage</u> (output elasticity wrt labor):**

Model $\phi_t(i_{it}, k_{it})$ non-parametrically to estimate α_L in (OP2)

  - **Example: use series estimator so we include polynomial terms in** i_{it}, k_{it}, $i_{it}*k_{it}$, $(i_{it})^2$, $(k_{it})^2$, etc.

- **<u>Second Stage</u> (to get output elasticity wrt capital, α_K)**

$q_{it} - \alpha_L l_{it} = \alpha_K k_{it} + g(\omega_{it-1}) + \xi_{it} + \nu_{it}$ **From Markov assumption [$\omega_{it} = g(\omega_{it-1}) + \xi_{it}$]**

$= \alpha_K k_{it} + g(\phi_{t-1} - \alpha_K k_{it-1}) + \xi_{it} + \nu_{it}$ **from eq (OP2)**

Estimate by Non-Linear Least Squares (NLLS)

RELATIONSHIP OF OLLEY-PAKES TO OTHER ESTIMATORS

- Can also be done in 1-step nonlinear GMM (Wooldridge, 2009).
- Relationship to Blundell-Bond (2000)
  - Two approaches are non-nested: (i) different timing of capital accumulation; (ii) underlying economic theory; (iii) modelling of ω_{it}
  - But BB model ω_{it} as AR(1), plus fixed effect plus error. OP (& proxy methods) do not allow for a fixed effect, but do allow a more general Markov process.
  - If BB model of ω_{it} is a good approximation, then OP nested as special case of BB (perfectly flexible labor, pre-determined capital).

SELECTION/SURVIVOR BIAS

- Large % of plants exit in most samples in a 5 year period
- Often researchers construct "balanced panel" using plants or firms that are active during entire period
- But balanced panel ignores selection:
  - Exiting firms tend to have low productivity (ω_{it}) draws
  - Capital-intensive firms more likely to stay even in presence of low ω_{it} draws.
- Will tend to cause upward bias on capital coefficient α_K
- OP use unbalanced panel & explicitly account for selection by conditioning expectation of ω_{it} *on survival*
  - Model exit probability non-parametrically and include as another control (strong assumption. Ideally we would have an external IV for exit)

KEY ASSUMPTIONS IN OP

1. **Strict monotonicity**
   - Investment function strictly monotonic in ω
2. **Scalar unobservable**
   - ω is the only econometric unobservable in investment equation
     - No unobserved input price variation across firms (except serially uncorrelated iid shocks to wages)
     - No structural unobservables affecting firm's optimal investment (e.g. efficiency of doing investment, heterogeneity of adjustment costs, etc.)
     - No optimization error in investment

SOME PROBLEMS WITH OLLEY-PAKES

- **Main Ones:**
  1. **Zero investment problem**
  2. **Exact multi collinearity/functional dependence**

- **Zero Investment problem**
  - Strict monotonicity of investment rule violated in micro data because many firms report zero investment
  - Solution? Levinsohn and Petrin (2003) suggest using different proxies to control for ω_{it} , e.g. Materials inputs. There are generally no zeros

SOME PROBLEMS WITH OLLEY-PAKES

- **Exact multi-collinearity problem (Ackerberg et al, 2015)**
  - Assume that markets are competitive & common & factor prices (wage = W, price = P); i.e. no exogenous firm specific input or output prices
  - First order conditions for labor demand implies no firm-level variation in labor, conditional on capital & ω

$$l_i = cons - \frac{1}{1-\alpha_L}(w-p) + \frac{\alpha_K k_i}{1-\alpha_L} + \frac{\omega_i}{1-\alpha_L}$$

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- **Solutions to multi-collinearity problem?**
  – Independent variation in factor prices (iid wages shocks after materials or *K* chosen but prior to *L* being chosen)
  – Optimization errors in labor (but <u>not</u> materials or inv!)
  – Adjustment cost for labor
  – Solutions are not very satisfactory

Estimating TFP heterogeneity

- Why estimate firm production functions?
- How to estimate productivity
- **Applications**

SOME EXAMPLE RESULTS

ALTERNATIVE ECONOMETRIC ESTIMATES OF THE PRODUCTION FUNCTION

	(1)	(2)	(3)	(4)
Estimation Method	OLS LEVELS	WITHIN GROUPS	OLLEY PAKES	GMM-SYS
Dependent variable:	Ln (Y) $_{it}$ sales	Ln (Y) $_{it}$ sales	Ln (Y) $_{it}$ sales	Ln (Y) $_{it}$ sales
ln (L) $_{it}$ labor	0.505 (0.020)	0.543 (0.022)	0.426 (0.022)	0.456 (0.064)
Ln(K) $_{it}$ capital	0.128 (0.013)	0.059 (0.015)	0.156 (0.036)	0.114 (0.050)
ln (N) $_{it,}$ materials	0.358 (0.017)	0.325 (0.022)	0.412 (0.024)	0.353 (0.046)
SC(1)p-value				0.000
SC(2) p-value				0.195
SARGAN p-value				0.153
SARGAN-DIF p-value				0.332
Firms	709	709	709	709

709 medium sized manufacturing firms, 1994-2004, UK,US, France and Germany

Source: Bloom and Van Reenen (2005), Appendix Table D1

SOME EXAMPLE RESULTS

- Coefficients in OLS levels close to CRTS (0.99)
- FE/WG coefficient on capital much smaller than other estimates (attenuation bias). OP gets highest on capital
- Diagnostics on GMM
  - Sargan-Hansen
  - LM Tests of Serial Correlation

PRODUCTIVITY OR MARK-UPS?

- Q meant to be output ("TFPQ"), but usually revenues deflated by industry price ("TFPR") firm prices unobserved.
  - Estimated coefficients mix technological parameters with price cost mark-ups

PRODUCTIVITY OR MARK-UPS?

- Q meant to be output ("TFPQ"), but usually revenues deflated by industry price ("TFPR") firm prices unobserved.
  - Estimated coefficients mix technological parameters with price cost mark-ups
- **Solutions:**
  - Get better data on firm prices (Foster, Haltiwanger and Syverson, 2009)
  - Make explicit the demand side & jointly estimate mark-ups.
- e.g. Klette & Griliches (1996) monopolistic competition: firm specific demand function: $q_i = -\eta(p_i - p_I) + d_I$
- p_i firm-specific log price (unobservable), p_I is industry log price (observable), d_I = industry demand shifters, η = the elasticity of demand.

PRODUCTIVITY OR MARK-UPS?

- Solving for firm prices $p_i = p_I - (q_i - d_I)/\eta$
- What we measure in data is real sales (r) not quantity (q)

$$r_i - p_I \equiv (q_i + p_i) - p_I$$

- Putting everything together, real sales equation is:

$$r_i - p_I = \left(1 - \frac{1}{\eta}\right)\left(\omega_i + \alpha_L l_i + \alpha_K k_i\right) + \frac{1}{\eta} d_I$$

- $1/\eta$ is the mark-up of price over marginal
- If we have demand shifters (e.g. Industry output) d_I then can get elasticity (η) & separate from productivity
- De Loecker (2011) generalizes this to OP set-up (multiproduct firms creates some extra variation)

USING PRODUCTION FUNCTION TO ESTIMATE MARKUPS

- De Loecker and Warzynski (2012): if we have consistent estimates of the output elasticity with respect to a variable factor (e.g. α_L), then can recover price-marginal cost markup:

$$\mu_i = \frac{\alpha_{L,i}}{s_{L,i}}$$

- Where s_L is share of variable factor in revenues (WN/PQ)
- Assumes no monopsony power over factor inputs (see Rubens, 2023)

SUMMARY

- Multiple ways to measure productivity
  - Accounting residual
  - Econometric estimation of production functions
- Hard to get at technological parameters, but much recent progress.
- Fundamental issue is that relying on “internal” rather than “external” IVs implies having to make more assumptions on properties of error terms
- Which exact method to use is context and question specific

BACK UP

- Dynamic Panel approaches usually consider state dependence – i.e. causal impact of lags, whereas production functions do no
- Need more observations per firm to implement Blundell Bond, etc.
  - Proxy variable methods need 2 or more adjacent periods per firm
  - Arellano & Bond (1991) needs at least 3 adjacent periods per firm (as uses t-2 as IV)
  - Blundell & Bond (2000) additional “levels” moments only increase efficiency if have at least 4 adjacent periods per firm
  - The greater the conditioning the more sample selection issues are a concern

1. RESIDUAL APPROACH - RELAXING ASSUMPTIONS

- **Example (3) Market Power (Hall, 1988)**
  - With market power factor, shares in revenue (s_L) smaller than α_L because of mark-up $s_L = \alpha_L/\mu$; $\mu_i = (P/c)_i$ = Price/marginal cost
  - *Monopolistic Competition*. η = price elasticity of demand). Q = industry quantity, P = industry price index

$$\frac{Q_i}{Q} = B\left(\frac{P_i}{P}\right)^{-\eta} \qquad s_L = \left(1 - \frac{1}{\eta}\right)\alpha_L$$

  - Use cost shares [WL/(WL+RK)] instead of revenue shares as these still equal to α_L. But now need to know R = user cost of capital
- Basic problem that we have to make a lot stronger assumptions to avoid econometrics

ARELLANO & BOND (1991) GMM - DETAILS

Consider simple dynamic model (easy to add in X's: labor capital)

$$\Delta q_{it} = \alpha \Delta q_{it-1} + \Delta v_{it}; i = 1, 2, ...N; t = 3, 4....T$$

No serial correlation in errors $E(v_{it} v_{i,t-1}) = 0$

GMM estimators for this model use instrument matrix of the for,

$$Z_i = \begin{bmatrix} q_{i1} & 0 & 0 & ... & 0 & ... & ... & 0 \\ 0 & q_{i1} & q_{i2} & & & & & \\ . & & & & & & & \\ 0 & 0 & 0 & ... & q_{i1} & ... & ... & q_{i,T-2} \end{bmatrix} \begin{matrix} \Delta v_{i3} \\ \\ \\ \Delta v_{iT} \end{matrix}$$

Rows correspond first difference equations for t =3,4,…T.
Exploits the moment conditions

$$E(Z_i ' \Delta v_i) = 0$$

$$\Delta v_i = (\Delta v_{i3}, \Delta v_{i4,} ....., \Delta v_{iT})'$$

ARELLANO & BOND (1991) GMM - DETAILS

Asymptotically efficient GMM estimator minimizes the criterion

$$J_N = \left(\frac{1}{N} \sum_{i=1}^{N} \Delta v'_i Z_i \right) W_N \left(\frac{1}{N} \sum_{i=1}^{N} Z'_i \Delta v_i \right)$$

Where the weight matrix is based on consistent estimators of first differenced residuals

$$W_N = \left(\frac{1}{N} \sum_{i=1}^{N} Z'_i \Delta \hat{v}_i \Delta \hat{v}'_i Z_i \right)^{-1}$$

One-Step weight matrix where H is T-2 square matrix

$$W_{1N} = \left(\frac{1}{N} \sum_{i=1}^{N} Z'_i H Z_i \right)^{-1} \qquad H = \begin{bmatrix} 2 & -1 & 0 & .. & 0 \\ -1 & 2 & -1 & .. & 0 \\ 0 & -1 & .. & -1 & 0 \\ .. & .. & .. & 2 & -1 \\ 0 & 0 & .. & -1 & 2 \end{bmatrix}$$

PRODUCTION FUNCTION APPLICATION OF BLUNDELL-BOND

$$q_{i,t} = \pi_1 q_{i,t-1} + \pi_2 l_{i,t} + \pi_3 l_{i,t-1} + \pi_4 k_{i,t} + \pi_5 k_{i,t-1} + \eta_i^* + v_{it}$$

"A-B" Moment conditions

$$E(x_{i,t-s} v_{it}) = 0; x_{it} = (y_{it}, l_{it}, k_{it})$$

When v_{it} ~ MA(0) s ≥ 2; When v_{it} ~ MA(1) s ≥ 3, etc.

"B-B" If we also assume fixed effects uncorrelated with changes in labor, capital and initial value added (Δy_{i2}) then we have further moments:

$$E[\Delta x_{i,t-1}(\eta_i^* + v_{it})] = 0$$

For v_{it} ~ MA(0). This means we can use lagged differences as instruments for the levels

ACF

- Value added production function (Leontief in materials)
- Use materials as proxy variable strict monotonicity, but give up on estimating OP first stage
- $m_{it} = f(\omega_{it}, k_{it}, l_{it})$
- $q_{it} = \alpha_L l_{it} + \alpha_K k_{it} + f^{-1}(k_{it}, m_{it}, l_{it}) + \nu_{it}$
- Call the predictions from this $\widehat{\Phi}$
- If we "guess" α_L and α_K we can calculate $\widehat{\omega}$ from $\widehat{\Phi}$ and $q_{it} = \alpha_L l_{it} + \alpha_K k_{it} + \omega_{it} + \nu_{it}$
- We can then get $\widehat{\xi}$ as residuals from regression of ω_{it} on $g(\omega_{it-1})$
- The sample analog to moment conditions can be used:
- $\frac{1}{N}\frac{1}{T}\sum_i \sum_t \left(\widehat{\xi_{it}}(\widehat{\alpha_L},\widehat{\alpha_K})k_{it}\right) = 0$ and $\frac{1}{N}\frac{1}{T}\sum_i \sum_t \left(\widehat{\xi_{it}}(\widehat{\alpha_L},\widehat{\alpha_K})l_{it}\right) = 0$
- To estimate $\alpha_{L,}\alpha_K$ by nonlinear GMM

COMPARE WITH OP

- Use investment as proxy variable. Given timing assumption on capital E($\widehat{\xi_{it}} k_{it}) = 0$
- $i_{it} = f(\omega_{it}, k_{it})$; $\varepsilon_{it} = \omega_{it} + \nu_{it}$
- $q_{it} - \alpha_L l_{it} = \alpha_K k_{it} + f^{-1}(k_{it}, i_{it}) + \nu_{it}$
- Call the predictions from this $\widehat{\Phi}$
- Guess α_K & calculate $\widehat{\omega}$ from $\widehat{\Phi}$ and $q_{it} - \alpha_L l_{it} = \alpha_K k_{it} + \omega_{it} + \nu_{it}$
- We can then get $\widehat{\xi}(\alpha_K)$ as residuals from reg of ω_{it} on $g(\omega_{it-1})$
- The sample analog to moment conditions can be used:
- $\frac{1}{N}\frac{1}{T}\sum_i \sum_t \left(\widehat{\xi_{it}}(\widehat{\alpha_K}) k_{it}\right)$ = 0 to estimate α_K

SOME EXAMPLE RESULTS

Table 3: Production Function Estimates from Pavcnik (2002)*

	OLS	Fixed Effects	Olley-Pakes
Unskilled Labor	0.178 (0.006)	0.210 (0.010)	0.153 (0.007)
Skilled Labor	0.131 (0.006)	0.029 (0.007)	0.098 (0.009)
Materials	0.763 (0.004)	0.646 (0.007)	0.735 (0.008)
Capital	0.052 (0.003)	0.014 (0.006)	0.079 (0.034)

* From Pavcnik, N. (2002) "Trade Liberalization, Exit, and Productivity Improvements: Evidence from Chilean Plants," *The Review of Economic Studies* 69, January, pp. 245-76

OTHER THINGS

- Value added equation vs. output equation.
- Standard approach is to assume gross output PF is Leontief in materials. Therefore, can regress output on L,K without materials
- Alternative is to estimate value added PF, so right hand side is VA = Output – materials. This was original OP set-up

OTHER PAPERS

- Gandhi, Navarro & Rivers (2021) Nonparametric approach
- Doraszelski and Jaumandreu (2018) Measuring bias of technical change